

LAB WORK NO. 1

CONVERSION AND OPERATIONS IN DIFFERENT NUMERATION BASES

1. Objective of the lab work

The purpose of this lab is to understand the way an integer or decimal number is converted from one base to another. This paper will focus on converting integer and decimal numbers from base 10 to a random base, especially bases 16, 2 and 8, and also on the reverse conversion, from a random base to base 10, especially from bases 16, 2 and 8. Direct conversion from base 16 to bases 2 or 8, and vice versa, will also be studied. Simple operations (add, subtract) in different numeration bases will be presented later.

2. Theoretical considerations

A numeration system consists of all the number representation rules by means of certain symbols, called digits.

For any numeration system, the number of distinct symbols for the system's digits is equal with the base (b). So, if the base $b=2$ (the numbers are written in binary), the symbols will be the 0 and 1 digits. For base $b=16$ (hexadecimal) the symbols will be 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. One can notice that for numbers written in a base higher than 10, other symbols (letters) are used besides the common digits for base 10. Therefore, in the case of numbers written in hexadecimal, the letters A, B, C, D, E, and F have the associated values of 10, 11, 12, 13, 14, and 15.

To make an easier distinction between numbers written in a certain base, at the end of the number another letter is written, which represents the base, for instance:

B for numbers written in binary (base 2)

Q for numbers written in octal (base 8)

D for numbers written in decimal (base 10)

H for numbers written in hexadecimal (base 16)

Usually, numbers written in decimal don't necessarily have to be followed by the „D” symbol, because this base is considered to be implicit.

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Further on, one can convert to bases 2 and 8 in the same manner, but there is also a quicker way to convert numbers between bases 2, 8 and 16, given the fact that for each hexadecimal digit there is a corresponding 4 binary digit number, and that for each digit in octal there are 3 corresponding binary digits, as seen in the table below:

| Value in decimal | Value in hexadecimal | Binary number corresponding to hexadecimal | Binary number corresponding to octal |
|------------------|----------------------|--|--------------------------------------|
| 0 | 0 | 0000 | 000 |
| 1 | 1 | 0001 | 001 |
| 2 | 2 | 0010 | 010 |
| 3 | 3 | 0011 | 011 |
| 4 | 4 | 0100 | 100 |
| 5 | 5 | 0101 | 101 |
| 6 | 6 | 0110 | 110 |
| 7 | 7 | 0111 | 111 |
| 8 | 8 | 1000 | |
| 9 | 9 | 1001 | |
| 10 | A | 1010 | |
| 11 | B | 1011 | |
| 12 | C | 1100 | |
| 13 | D | 1101 | |
| 14 | E | 1110 | |
| 15 | F | 1111 | |

There also has to be considered that when a number is passed between bases 2, 8, 16, grouping the digits is done „from the comma towards the extremities”, meaning that for integer numbers it is done from the right to the left (by completing the number with zeros on the left side in case this is required, that is on the side that doesn't affect its value), and for decimal numbers the grouping is done after the comma, from the left side to the right, by filling in with zeros at the right side of the number.

Example:

- 347 (D) = 15B (H) = 1 0101 1011 (B) = 533 (O)
- 438 (D) = 1B6 (H) = 1 1011 0110 (B) = 666 (O)

2.1.2. Converting the decimal part

To convert a sub-unitary number (that is the fractional part of a number) from base 10 to a random base, a series of successive multiplications of the fractional parts has to be done, until the fractional part becomes null, a period is found or the representation capacity is surpassed (a sufficient number of digits has been obtained, although the algorithm hasn't finished). The digit that surpasses the decimal part at each multiplication is a digit of that number, in the base that the conversion is made to.

As an example, it is easy to use the following scheme, in which the digits of the representation are more clearly represented with the help of the two lines. Also, the position of the comma is more obvious (the numbers below the first multiplication, that is the ones under the line, are positioned after the comma). One must notice that only what is on the right side of the comma is multiplied.

Example:

- Conversion of the number 0,47 (D) to binary, octal and hexadecimal:

| | |
|----|------|
| 0, | 47*2 |
| 0 | 94 |
| 1 | 88 |
| 1 | 76 |
| 1 | 52 |
| 1 | 04 |
| 0 | 08 |
| 0 | 16 |
| 0 | 32 |
| 0 | 64 |
| 1 | 28 |
| 0 | 56 |
| 1 | 12 |
| 0 | 24 |
| 0 | 48 |
| 0 | 96 |
| 1 | ... |

So: 0,47 (D) = 0,0111 1000 0101 0001 (B) = 0,7851 (H) = 0,3605 (O)

Converting a number that has both an integer and a decimal part is done by successively converting the integer part and the decimal one.

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Example:

- Representation in bases 2 and 16 of the real number 14, 75
We obtain: 14 (D) = 1110 (B) = E (H)
and 0, 75 (D) = 0, 11 (B) = C (H).
Therefore 14, 75 (D) = 1110, 11 (B) = E, C (H)

2.2. Converting a number from a random base to base 10

For converting a number from a random base to base 10 one can use the formula that was defined in the first part of this lab, meaning that if a number is written in a random base „b” as an integer and decimal part:

$Nr(b) = C_n C_{n-1} C_{n-2} \dots C_2 C_1 C_0, D_1 D_2 D_3 \dots$

than its value in base 10 will be:

$$Nr(10) = C_n * b^n + C_{n-1} * b^{n-1} + \dots + C_2 * b^2 + C_1 * b^1 + C_0 * b^0 + \\ + D_1 * b^{-1} + D_2 * b^{-2} + D_3 * b^{-3} + \dots$$

Example:

- The number 3A8 (H) is given in hexadecimal and its value in decimal is required:
 $N = 3 * 16^2 + 10 * 16^1 + 8 = 3 * 256 + 160 + 8 = 936$ (D)
- The fractional number 0, 341 (Q) written in base 8 is given and its value in decimal is required:
 $N = 3 * 8^{-1} + 4 * 8^{-2} + 1 * 8^{-3} = 3/8 + 4/64 + 1/512 = 0.4394$ (D)
- The binary number 110, 11 (B) is given and its values in hexadecimal and decimal are required:
 $N = 110, 11$ (B) = 6, C (H) = 6, 75 (D)

2.3. Simple operations with numbers written in different bases

Further, the adding and subtracting of integer unsigned numbers written in binary, octal and hexadecimal is presented.

2.3.1. Addition

Adding is done by the same rules that exist in decimal, with the remark that the highest digit in a base „b” will be b-1 (9 in decimal, 7 in octal, 1 in binary and F in hexadecimal). Therefore, when a result higher

than $b-1$ is obtained by adding two digits of rank „ i ”, a carry will appear to the digit having the „ $i+1$ ” rank. On the position ranked „ i ” will be the rest of the division of the result obtained by adding the rank „ i ” digits, by the base. The carry to the „ $i+1$ ” rank digit will become a new unit to the adding of the rank „ $i+1$ ” digit, meaning that a 1 will be added (the carry).

Examples:

| | | |
|---------------|-----------|-------------|
| 1111 1 | 11 | 11 |
| 01010110 (B)+ | 1364 (Q)+ | 6D8A32 (H)+ |
| 10110101 (B) | 3721 (Q) | 33E4C8 (H) |
| 100001011 (B) | 5305 (Q) | A16EFA (H) |

The 1 unit carry to the higher ranked digit was marked by writing a 1 above the superior ranked digit to which the carry was made. The adding operation in binary is useful for representing a number in complement by 2, when one chooses to add a 1 to the number’s representation in complement by 1 (see lab about data representation).

Examples:

- The students have to add the 2 integer numbers 347 (D) and 438 (D) that were converted earlier in the lab to bases 16 and 8, and to check the result by converting it to base 10.

347 (D)+ 438 (D) = 785 (D)

15B (H) + 1B6 (H) = 311 (H). Checking: $311 (H) = 3*256+1*16+1 = 785$

533 (Q) + 666 (Q) = 1421 (Q). Checking: $1421 (Q) = 1*512+4*64+2*8+1 = 785$

2.3.2. Subtraction

The subtraction rules in decimal are also used for other bases: if one can’t subtract two digits of rank „ i ” (if the minuend’s digit is lower than the subtrahends), then a unit is „borrowed” from the next ranked digit („ $i+1$ ”). If the digit from which borrowing is desired is 0, than one borrows further from the next ranked digit.

Examples:

| | |
|--|--|
| $\begin{array}{r} \cdot\cdot \\ 01011010 (B) - \\ 01001100 (B) \\ \hline 00001110 (B) \end{array}$ | $\begin{array}{r} \cdot\cdot \\ A3D4 (H) - \\ 751B (H) \\ \hline 2EB9 (H) \end{array}$ |
|--|--|

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- The students have to subtract the two integer numbers 347 (D) and 438 (D) that were converted earlier in the lab to the numeration bases 16 and 8, and to check the result by converting it to decimal.

$$438 - 347 = 91 \text{ (D)}$$

$$1B6 \text{ (H)} - 15B \text{ (H)} = 5B \text{ (H)}. \text{ Checking: } 5B \text{ (H)} = 5 \cdot 16 + 11 = 91$$

$$666 \text{ (Q)} - 533 \text{ (Q)} = 133 \text{ (Q)}. \text{ Checking: } 133 \text{ (Q)} = 1 \cdot 64 + 3 \cdot 8 + 3 = 91$$

Subtraction is useful when it is desired to represent numbers in complement by 2 and a subtraction from $2^{\text{no_of_reprez_digits} + 1}$ of the absolute value of the number is made.

3. Lab tasks

1. Convert integer numbers from base 10 to bases 2, 8 and 16
2. Convert integer numbers between bases 2, 8 and 16
3. Convert integer numbers from bases 2, 8 and 16 to base 10
4. Convert real numbers to bases 2 and 16
5. Add and subtract numbers in bases 2 and 16

Notes