CHAPTER 4

OPTICAL SPECTRAL FILTERS AND GRATINGS

4.1 INTRODUCTION

The function of an electronic or passive \((LRC)\) filter is to recognize a narrow band of electrical frequencies from a multiplicity and either pass it or reject it. Optical spectral filters function the same way, and they are key components in optical transmission systems. Optical spectral filters are based on interference, diffraction, or absorption, and they are used in fixed and in tunable filters.

In this chapter, we examine the following filters:

- The Fabry–Perot interferometer
- The Bragg reflector
- Dielectric thin-film interference
- The acousto-optic tunable filter
- The absorption filter
- The hybrid filter

4.2 FABRY–PEROT INTERFEROMETER

The Fabry–Perot interferometer is based on the interference of multiple reflections of a light beam by two surfaces of a thin plate (Figure 4.1). The condition for interference maxima for each wavelength is \(2d \sin \Theta = n \lambda\), where \(n\) is an integer and \(d\) is the thickness of the plate. Clearly, this condition is satisfied by a number of wavelengths that are multiples of \(2\pi\), and at the maxima points the intensity is \(I_{\text{max}} = E^2\).
If the reflectivity of the plate surfaces is $R$, then the intensity between maxima is $I = \frac{(1 - R)^2}{(1 + R)^2}$. As the reflectivity of the surface increases, the intensity between maxima decreases, thus increasing the sharpness of the interferometer (Figure 4.2).

**4.2.1 Fabry–Perot Resonator**

The *Fabry–Perot resonator* is an arrangement of two parallel plates that reflect light back and forth. To examine how this functions, consider the following (Figure 4.3):

- Let two plane parallel semitransparent reflectors (half-mirrors) be separated by $d$.
- Let there be a medium with attenuation $\alpha_s$ and gain $g$.
- Let $R_1$ and $R_2$ be the *power-reflected coefficients* for reflectors 1 and 2.
- Let a pulse of photons $E(t,x)$ enter reflector 2, $E(t = 0, x = 0)$. 
Then,

\[ E(t,x) = A \exp\left\{-\frac{\alpha_s x}{2}\right\}\left[j(\omega t - \beta x)\right] \],

where \( \alpha_s \) is the intensity attenuation coefficient, \( \beta \) is the propagation constant, and \( \omega \) is the light frequency. A typical Fabry–Perot resonator may have the plates fixed, in which case it is known as an etalon, or adjustable with a micrometer: an interferometer.

The key question in this arrangement is: What is the condition for resonance? The field at mirror 2 \((x = 0)\), after it has been reflected by mirrors 1 and 2, is

\[ E(t,0) = R_1R_2A \exp\{[(g-\alpha_s)d][j(\omega t-2\beta d)]\} \]

where \( g \) is the intensity gain coefficient, the first exponent is the propagation component, and the second is the phase component.

For steady-state oscillations, the amplitude of the initial light pulse \((t = 0, x = 0)\), must be equal to the amplitude after it has been reflected back and forth. This leads to two conditions: The amplitude condition

\[ R_1R_2A \exp\{[(g-\alpha_s)d]\} = A \]

and the phase condition

\[ \exp(-j2\beta d) = 1. \]

The phase condition is satisfied only if

\[ 2\beta d = 2\pi m; \quad \beta = \frac{2\pi n}{\lambda}, \]

where \( m \) is an integer, \( n \) is the refractive index, \( \beta \) is the propagation constant, and \( \lambda \) is the wavelength in free space.

The values of \( \lambda \) that satisfy the relationship

\[ \lambda = \frac{2dn}{m} \]

provide the resonant wavelengths or modes of the Fabry–Perot resonator.

**Example**

For \( m = 1 \) and \( n = 1 \), then \( \lambda = 2d \); for \( m = 2 \) and \( n = 1 \), then \( \lambda = d \).
The frequency spacing $\Delta f$ between consecutive (longitudinal) modes is obtained from

$$m - (m - 1) = \frac{zd}{c}f_m - \frac{zd}{c}f_{m-1}$$

or

$$1 = \frac{2zd}{c} \Delta f,$$

from which one derives

$$\Delta f = \frac{c}{2zd}$$

and

$$\Delta \lambda = \frac{\lambda^2}{2zd}.$$

The latter relation indicates that a multiplicity of frequencies (wavelengths) is transmitted through the Fabry–Perot resonator. A typical transmittance profile is shown in Figure 4.4.

![Typical transmittance profile of a Fabry–Perot resonator.](image)

**4.2.2 Finesse**

An indication of how many wavelength (or frequency) channels can simultaneously pass without severe interference among them is known as the *finesse* of the Fabry–Perot resonator. Finesse is a measure of the energy of wavelengths within the cavity relative to the energy lost per cycle. Thus, the higher the finesse, the narrower the resonant line width. The finesse is equivalent to the $Q$-factor of electrical filters.

Cavity losses due to imperfections of mirrors (e.g., flatness) and angle of incidence of the light beam impact the finesse value. Usually, the finesse is dominated by mirror losses due to power that flows in and out as a result of the semitransparency of the mirrors. Clearly, if the mirrors were fully reflective, they would not allow light in or out, and if they were only slightly reflective, the cavity would not sustain an adequate amount of light power in it.

Assuming the same reflectivity $R$ for both mirrors, the finesse $F$ is expressed by

$$F = \frac{\pi R}{2(1 - R)}.$$

Finesse values of 20–100 have been achieved.
4.2.3 Spectral Width, Line Width, and Line Spacing

Spectral width is defined as the band of frequencies through which a filter will pass (Figure 4.5). The spectral width is characterized by an upper and lower frequency (wavelength) threshold. In addition, it is characterized by a gain curve that is a measure of the degree of flatness over the spectral width.

![Figure 4.5 Definition of spectral width, line width, and line spacing.](image)

Line width or channel width is the width of the frequency channel. An ideal channel would be monochromatic; that is, a single wavelength. However, this is not possible, and thus the line width is a measure of how close to an ideal channel is, as well as an indication of the spectral content of the channel.

Line spacing is defined as the distance in wavelength units (nm) or in frequency units (GHz) between two channels.

These definitions are important in optical system design. They determine the number of channels the system can support, as well as the distance for error-free communication over a dispersive fiber.

4.2.4 The Fabry–Perot Filter

The Fabry–Perot filter consists of two high-reflectance multilayers separated by a λ/2 space layer (Figure 4.6). Multiple interference in the space layer causes the filter output spectral characteristic to peak sharply over a narrow band of wavelengths that are multiples of the λ/2 spacer layer. Thus, the Fabry–Perot interference filter is used exclusively as a band-pass filter.
4.3 **BRAGG GRATING**

A Bragg grating is an arrangement of parallel semireflecting plates. To examine how this functions, consider the following (Figure 4.7):

- Let $N$ (periodic) plane parallel semireflectors be separated by $d$ (Bragg spacing).
- Let there be a medium with attenuation $\alpha_s$ and gain $g$.
- Let $R$ be the *power-reflected coefficient*, such that $R << 1$, and $T$ the *power-transmitted coefficient*, such that $T \sim 1$.
- Let a pulse of photons $E(t,x)$ enter through mirror 1, $E(t=0,x=0)$.

Then

$$E(t,x) = A \exp \left\{ \frac{-\alpha_s x}{2} [j(\omega t - \beta x)] \right\},$$

where $\alpha_s$ is the intensity attenuation coefficient, $\beta$ is the propagation constant, and $\omega$ is the light frequency. The key question in this arrangement is: *What is the condition for strong reflection of a particular frequency?*

Let the first light pulse at the first mirror be $E_{in}$. The reflected part of $E_{in}$ by the first semitransparent mirror is $E_{R1}(0) = E_{in}(0)R$ and the transmitted part is $E_{T1}(0) = E_{in}(0)T$. $R$ and $T$ are the reflection and transmission coefficients, respectively.
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The transmitted portion $E_{T_1}(0)$ is partially reflected [$E_{R_2}(d) = E_{T_1}(0)$ $R \exp(-j\beta d)$] and partially transmitted [$E_{T_2}(d) = E_{T_1}(0) T \exp(-j\beta d)$] by the second mirror, and so on. The reflected part at the $N$th mirror is

$$E_{RN}[(N-1)d] = E_{T(N-1)} [(N-2)d] R e^{-j\beta d}$$

and the transmitted part is

$$E_{TN}[(N-1)d] = E_{T(N-1)} [(N-2)d] T e^{-j\beta d}.$$

The part reflected by the $N$th mirror arrives back to the first mirror. Taking into account the propagation constant, this is

$$E_{RN}[(N)d] = E_{T(N-1)} [(N-2)d] g^{N-1} e^{(N-1)j\beta d}.$$

4.3.1 Bragg Reflector

To derive the condition for total reflection, one examines the sum of all reflected components at the first mirror. The sum yields a geometric series from which the relationship is obtained:

$$E_{R_{tot}}(0) = E_{in}(0) \frac{R(1-M^N)}{1-M},$$

where $M = T^2 \exp(-2j\beta d)$. That is, if $N$ is sufficiently large, then the total reflected energy at the first mirror approximates the incident energy, even if $R$ is small.

4.3.2 The Bragg Condition

In the preceding relationship, the phase angle of the waves at each mirror was arbitrary. If the phase angle could be a multiple of $2\pi$, a condition for strong reflection could be obtained; that is,

$$\arg(M) = 2 \arg T - 2\beta d = 2 \arg T - 2\left(\frac{2\pi}{\lambda}\right) d = 2\pi n,$$

where $n$ is an arbitrary number.

For simplicity, when phase = 0 or setting $\arg(.) = 0$, then $-(2\pi/\lambda)d = n\pi$, from which the condition for strong reflection or Bragg condition is

$$d = -\frac{n\lambda_B}{2}.$$ 

That is, the Bragg spacing (or grating period) should be an integer multiple of the half-wavelength. The negative sign denotes reflection, and $n$ denotes the order of the Bragg grating. When $n = 1$ (first order) then $d = \lambda/2$, and when $n = 2$ (second order) then $d = \lambda$.

4.4 FIBER BRAGG GRATINGS

A fiber Bragg grating (FBG) consists of a fiber segment whose index of refraction varies periodically along its length. Variations of the refractive index constitute discontinuities that emulate a Bragg structure. A periodic variation of the
refractive index is formed by exposing the germanosilicate core of the fiber to an intense ultraviolet (UV) optical interference pattern that has a periodicity equal to the periodicity of the grating to be formed. When the fiber is exposed to the intense UV pattern, structural defects are formed and thus a permanent variation of the refractive index having the same periodicity with the UV pattern. Figure 4.8a illustrates an FBG using the UV method and Figure 4.8b illustrates a monolithically made FBG with a corrugated In$_x$Ga$_{1-x}$As$_y$P$_{1-y}$ over an InP substrate. A fiber Bragg grating incorporated in-line with a transmitting fiber is also called an “in-fiber Bragg grating.”

![UV interference pattern](image)

**Figure 4.8** A Bragg grating made by exposing a fiber with (a) a UV pattern and (b) a monolithic one.

The interference pattern has a periodicity that depends on the wavelength band in which the FBG is designed to operate. For near-infrared wavelengths of about 1.55 µm, the grating is made with a periodicity $d$ of 1–10 µm.

The grating reflectivity, $R$, for a given mode at center wavelength is given by

$$R = \tanh^2 \left[ \frac{\pi L \Delta n \eta(V)}{\lambda_b} \right],$$

where $L$ is the length of the grating, $\Delta n$ is the magnitude of index perturbation, and $\eta(V)$ a function of the fiber parameter $V$ that represents the fraction of the integrated mode intensity contained in the core.

This grating assumes a single-mode polarization-maintaining fiber with a circular core. However, when the core is (imperfectly) elliptical, the grating supports two propagation modes, and each polarization axis has a different propagation constant.
The UV pattern is formed by means of one of several optical methods (diffraction or interferometry) that generate an interference pattern of alternating minima and maxima of light intensity. Regardless of the method used, the interference pattern must be of high quality, with uniform periodicity, high contrast, and sharp edges.

The UV source is provided by an excimer laser that operates at a wavelength in the 157–351 nm range. The peak absorption is at 240 nm, and thus this wavelength is the most efficient. Excimer lasers may produce hundreds of millijoules in a 10–40 ns pulse and can create the grating pattern in a fiber in a single high-energy shot, as the fiber is drawn (see Section 3.2.1). Continuous wave (CW) laser sources at 1-W output may also be used.

Applying periodic pressure along the fiber may also form an FBG. Pressure also alters the structure of the fiber and the refractive index, thus creating an FBG.

There are many uses for FBGs. For example, an FBG placed at the output of a circulator reflects back only the wavelength it is designed for, thus constructing a bandstop filter. Placed at the output of a laser, the FBG reflects back portion of the transmitted power to be monitored by a light-emitting diode (LED). When the laser ceases to function, the LED detects it and sends a message to the system.

A similar Bragg grating reflector, based on a stacked-dielectric structure, is composed of quarter-wavelength-thick layers known as a photonic lattice, each with a different refractive index. Photonic lattice reflectors have been found to reflect wavelengths over all possible angles of incidence, and unlike mirror-based reflectors, they do not absorb incident energy.

Fiber Bragg gratings with a linearly variable pitch may compensate for chromatic dispersion, known as chirped FBGs (Figure 4.9). In this case, because of the linearly varying pitch (or chirp), wavelengths within a channel are reflected back at different depths of the grating, thus compensating for the travel time variation (see Section 3.12) of wavelengths in a channel. Thus, chirped FBGs perform chromatic compression on a chromatically dispersed pulse.

Figure 4.9 A fiber Bragg chirped grating reflects dispersed wavelengths of a channel at different depths, thus compensating for the spectrally dispersed width.
4.5 TUNABLE BRAGG GRATINGS

The wavelength at which the reflection is maximal is given by the Bragg condition:

\[ \lambda_B = \frac{2d}{n} \]

To make the Bragg grating tunable (i.e., to control the reflected wavelength), the Bragg spacing (grating period) must be controllable. This is achieved by one of several methods. For example, application of a stretching force elongates the fiber, which thus changes its period (*mechanical tuning*). Application of heat elongates the fiber, thus changing its period (*thermal tuning*).

FBGs final applications in fiber dispersion compensation, in gain flattening of erbium-doped fiber amplifiers (EDFAs), and in add–drop multiplexers/demultiplexers. However, the fiber used to make an FBG should be free of imperfections as well as of microscopic variations of the refractive index.

4.6 DIELECTRIC THIN FILM

*Dielectric thin-film* (DTF) interference filters consist of alternating quarter-wavelength-thick layers of high refractive index and low refractive index.

Light reflected within the layers of high index does not shift its phase, whereas light within the low index shifts by 180°. Taking into account the travel difference (in multiples of \(2 \times \lambda/4\)), the successive reflections recombine constructively at the front face, producing a highly reflected beam with wavelengths within a narrow range (Figure 4.10). Outside this range the output wavelengths drop abruptly.

The primary considerations in DTF design are:

- Low-pass-band loss \(< 0.3 \, \text{dB}\)
- Good channel spacing \(> 10 \, \text{nm}\)
- Low interchannel cross-talk \(> -28 \, \text{dB}\)

![Figure 4.10 A dielectric thin-film interference filter made with alternate layers of high and low refractive index; each layer is a quarter-wavelength thick.](image)
The wavelength range at the output of the $\lambda/4$ stack depends on the ratio of high to low refractive index. Thus, a DTF can be used as a high-pass filter, a low-pass filter, or a high-reflectance layer.

### 4.7 POLARIZING BEAM-SPLITTERS

Polarizing beam-splitters, in general, consist of two prisms connected with a polarizing film. An incident circularly polarized beam of light is separated in two linearly polarized beams in one reflected and another refracted one in transverse equipment (TE) and the other in transverse multiplexer (TM) mode (Figure 4.11).

![Figure 4.11 A polarizing beam-splitter consisting of two prisms and a polarizing film.](image)

### 4.8 TUNABLE OPTICAL FILTERS

_Tunable optical filters_ (TOFs) may be constructed with passive or active optical components. The salient characteristic of TOFs is their ability to select the range of filtered wavelengths. To be useful in optical communication systems, however, they must satisfy certain requirements:

- Wide tuning range
- Constant gain
- Narrow bandwidth
- Fast tuning
- Insensitivity to temperature (no frequency drift with temperature variations)

In the following sections, we examine some TOFs.
4.9 ACOUSTO-OPTIC TUNABLE FILTERS

Acousto-optic tunable optical filters (AOTFs) are based on the Bragg principle; that is, the only wavelengths that pass through the filter are those that comply with the Bragg condition. The index of refraction is made to fluctuate periodically by applying a radio-frequency (RF) acoustical signal to an optically transparent waveguide. The applied acoustical frequency (vibrations) on the waveguide disturbs its molecular structure with a certain periodicity, which determines the periodicity of the index of refraction. The polarization of the optical wavelength that complies with the Bragg condition is also rotated from TE to TM.

An AOTF consists of an acousto-optic TE-to-TM converter [a surface acoustic wave device (SAW), on which the acoustic signal is applied], two crossed polarizers, and two optical waveguides in very close proximity, such that light may be coupled from one waveguide to the other (Figure 4.12).

![Figure 4.12 Concept of an acousto-optic tunable filter.](image)

The selected \(\lambda\) is related to the applied acoustic frequency \(f_a\) (in MHz) by

\[
\lambda = \frac{(\Delta n)V_a}{f_a},
\]

where \(\Delta n\) is the medium birefringence for the selected \(\lambda\), and \(V_a\) is the acoustic velocity in the waveguide medium. Similarly,

\[
\lambda = \Lambda(\Delta n),
\]

where \(\Lambda\) is the wavelength of the acoustic wave.

The acoustic power \(P_a\) for 100% polarization rotation of the selected \(\lambda\) is

\[
P_a = \frac{\lambda^2 A}{2L^2 M_2} \text{ (mW)},
\]

where \(A\) is the acoustic transducer area, \(L\) is the transducer width, and \(M_2\) the acoustic figure of merit of the medium. \(P_a\) is expressed in hundreds of milliwatts.

The filter pass-band \(\Delta \lambda\) is approximated by

\[
\Delta \lambda = \frac{0.8 \lambda^2}{L \Delta n} \text{ (\mu m)}.
\]

The access time \(\tau\) is estimated by

\[
\tau = \frac{L}{V_a} \text{ (\mu s)}.
\]
The TE-to-TM converter of an AOTF filter is constructed with Ti:LiNbO₃ on which the remaining components may also be integrated to produce a single and compact component.

AOTFs are either collinear or noncollinear. Collinear AOTFs are those in which the optical signal propagates and interacts collinearly with the acoustic wave. Otherwise, they are noncollinear.

AOTFs may also be polarization independent. Such devices consist of a complex acousto-optic structure that selects a wavelength regardless of its polarization state.

The salient characteristics of AOTF filters (typically of the collinear type) are:

- Broad tuning range (1.2–1.6 μm)
- Narrow filter bandwidth (< 1 nm)
- Fast tunability (~ 10 μs)
- Acceptable insertion loss (< 5 dB)
- Low cross-talk (< –20 dB)
- Selection of one to many wavelengths
- Possible wavelength broadcast
- Easy wavelength registration and stabilization

AOTFs are used as:

- Single-wavelength tunable receivers
- Multiwavelength tunable receivers
- Wavelength-selective space switch (demultiplexers)

A disadvantage of the typical AOTF is misalignment of the polarization state of incoming light. Although the direction of the polarizer is known, the polarization state of incoming light is hard to maintain in a single-mode fiber and thus hard to control. Polarization mismatch results in coupling loss.

Another disadvantage of the typical AOTF is the frequency shift of light by an amount equal to the acoustical frequency due to Doppler effect. However, devices have been constructed that counterbalance the Doppler effect.

4.10 THE MACH–ZEHNDER FILTER

The Mach–Zehnder filter is based on the interference of two coherent monochromatic sources that are based on the length difference, and thus the phase difference, of two paths (see Section 2.3.9), thus contributing positively or negatively.
In fiber-optic systems, a phase difference between two optical paths may be artificially induced. Consider an input fiber with two wavelengths $\lambda_1$ and $\lambda_2$. The optical power of both wavelengths is equally split (directional coupler 1), and each half is coupled into a waveguide, one of which is longer than the other $L$ and $(\Delta L)$. The two halves arrive at a second directional coupler or combiner at different phases and, based on the phase variation and the position of the output fiber, each wavelength interferes constructively on one of the two output fibers and destructively on the other. That is, wavelength $\lambda_1$ interferes constructively on the first fiber and wavelength $\lambda_2$ on the second (Figure 4.13).

![Figure 4.13 The principle of a Mach–Zehnder filter.](image)

This arrangement is used to construct an integrated device that functions as a filter or as a wavelength separator, known as a *Mach-Zehnder filter*, according to which two frequencies at its input port are separated and appear at two output ports. A more detailed description of the Mach-Zehnder filter follows.

A mix of two wavelengths arrives at coupler 1. Coupler 1 equally distributes the power of wavelengths $\lambda_1$ and $\lambda_2$ into two waveguides having an optical path difference $\Delta L$. Because of the path difference, the two waves arrive at coupler 2 with a phase difference

$$\Delta \phi = \frac{2\pi f (\Delta L) n}{c},$$

where $n$ is the refractive index of the waveguide. At coupler 2, the two waves recombine and are directed to two output ports. However, each output port supports the one of the two wavelengths that satisfies a certain phase condition.

Wavelength $\lambda_1$ is obtained at output port 1 if the phase difference at the end of $L$ satisfies the conditions

$$\Delta \phi_1 = (2m - 1)\pi,$$

that is, where $\lambda_1$ contributes maximally. Wavelength $\lambda_2$, is obtained at port 2 if the phase difference satisfies the condition...
\[ \Delta \Phi_2 = 2m\pi, \]

that is, where \( \lambda_2 \) contributes maximally; \( m \) is a positive integer. Then

\[ \frac{2\pi f_1 \Delta L n}{c} = (2m - 1)\pi, \quad \text{and} \quad \frac{2\pi f_2 \Delta L n}{c} = 2m\pi. \]

These relationships are satisfied for a number of \( m \) values. Thus, this filter exhibits periodic pass bands.

From the last two relations, the optical channel spacing \( \Delta f \) is derived:

\[ \Delta f = \frac{c}{2n(\Delta L)}. \]

### 4.10.1 Tunability of the Mach–Zehnder Filter

If the quantity \( \Delta L \) can be adjusted at will, it is clear that the Mach–Zehnder filter can be tuned. The purpose of the quantity \( \Delta L \) is to introduce the desired phase shift at the entry point of directional coupler 2. Thus, the phase shift is controlled by controlling the propagation delay of the path \( L + \Delta L \) with respect to path \( L \). This is accomplished either by altering the refractive index of the path (and thus the effective optical path), by altering its physical length, or by both means.

The phase may be controlled by one of several methods (Figure 4.14):

- Mechanical compression, by means of a piezoelectric crystal, alters the physical length of the waveguide segment and its refractive index.
- Certain optical materials alter their refractive index when exposed to heat; a thin-film thermoelectric heater placed on the longer path would control the refractive index of the path. A polymer material known to change its refractive index when exposed to heat is per-fluoro-cyclo-butane (PFCB).

![Figure 4.14](image)

**Figure 4.14** A Mach–Zehnder filter can be tuned by controlling the temperature of the \( L + \Delta L \) path.
Thus, by controlling the refractive index of the path, the phase on the effective optical path \( L + \Delta L \) is controlled and the wavelength selectability of the device is accomplished, making the Mach–Zehnder filter a tunable optical frequency discriminator (OFD).

The Mach–Zehnder filter may also be cascaded to construct a multilevel filter (Figure 4.15). For example, eight wavelengths, \( \lambda_1-\lambda_8 \), are separated by one filter into two groups, \( \lambda_1, \lambda_3, \lambda_5, \lambda_7 \) and \( \lambda_2, \lambda_4, \lambda_6, \lambda_8 \). Each group is separated by two more filters into four subgroups (\( \lambda_1, \lambda_5 \)), (\( \lambda_3, \lambda_7 \)), (\( \lambda_2, \lambda_6 \)), and (\( \lambda_4, \lambda_8 \)), and so on, until all the wavelengths have been separated.

### 4.11 ABSORPTION FILTERS

Absorption filters consist of a thin film made of a material (e.g., germanium) that exhibits high absorption at a specific wavelength region. Their operation depends heavily on material properties, and thus there is little flexibility in making the absorption edges of the material very sharp.

However, when absorption materials are used in combination with interference filters (DTF, Fabry–Perot), a filter is produced that combines both sharp rejection edges and interference filter flexibility.

![Figure 4.15 The Mach–Zehnder filter may be cascaded to construct a multilevel filter.](image)

### 4.12 BIREFRINGENCE FILTERS

The properties of birefringent crystals may be used to construct optical tunable filters. Consider two quarter-wave birefringent disks positioned in parallel and such that the first disk has its fast axis at \(+45^\circ\) and the second at \(-45^\circ\) (Figure 4.16a).

Based on this, the retardation to a monochromatic beam propagating in the \( z \) axis is summed up to zero because one disk accelerates as much as the other decelerates. If one of the two disks is rotated by an angle \( 45^\circ + \rho \), then an acceleration or a deceleration that is proportional to the angle \( \rho \) is introduced, and a phase-controlling mechanism has been constructed. However, in this two-disk structure, as one of the two disks rotates, it also rotates the polarization of the beam. This is rectified by having the
two 45° disks fixed and inserting between them a disk that can rotate (Figure 4.16b). This arrangement constructs a single tuning stage.

Now, if the beam prior to entering the first disk has been split by a birefringent crystal into an ordinary and an extraordinary ray, then these rays are controlled differently and, as they recombine at the output, a wave-tuning mechanism or a tunable filter is constructed. This type of filter can be expanded to include more levels of tuning stages.

4.13 HYBRID FILTERS

Hybrid filters consist of a structure that combines different filter types and other optical components, such as a DTF and a grating. Hybrid filters take advantage of the grating filter's ability to separate closely spaced optical wavelengths and of the DTF filter's ability to separate widely spaced optical wavelengths.

4.14 TUNABLE FILTERS: COMPARISON

Each tunable filter has its own performance characteristics. Therefore, depending on the application, a filter type that best matches the performance requirements should be used.

If a large number of channels (~100) is required, Fabry–Perot and acousto-optic filters are better suited than electro-optic and semiconductor filters, which can process fewer channels (~10).

If fast tuning (~ns) is required, electro-optic and semiconductor filters are better suited than acousto-optic (~μs) and Fabry–Perot (ms) devices, although Fabry–Perot filters that use liquid crystals may also tune fast (in the microsecond range).

Mechanical tuning is slow (1–10 ms) but has a wide tuning range (~500 nm) compared with acousto-optic tuners (~250 nm) and electro-optic tuners (~16 nm).

If low loss is required, then semiconductor filters exhibit negligible loss compared with other types, which exhibit a loss in the order of 3–5 dB.

Based on this, the selection of the filter type in communications depends on the application and service the system is designed for. In applications with a large number of channels but relatively slow switching speeds (e.g., video broadcasting), the Fabry–Perot seems to be better suited. In applications with few but very fast switching times (circuit-switching of a few channels), the electro-optic or semiconductor...
type is better suited. In addition, the tuning range and the cost of each type should be taken into account, particularly in systems that require a large number of components or are cost sensitive.

### 4.15 DIFFRACTION GRATINGS

A diffraction grating is an arrayed slit device that takes advantage of the diffraction property of light and reflects light in a direction that depends on the angle of incident light, the wavelength, and the grating constant (Figure 4.17). That is, a diffraction grating reflects wavelengths in different directions when a mixed-wavelength beam impinges on it (see also Section 2.3.7 and Figure 2.7).

![Figure 4.17 A diffraction grating is characterized by the blaze angle and the number of slits per unit length.](image)

The blaze angle and the number of slits per unit length characterize a diffraction grating. For a given center wavelength $\lambda$, the blaze angle $\theta_B$ is set at

$$\theta_B = \sin^{-1} \left( \frac{\lambda}{2d} \right).$$

When the reflected light has a path difference equal to $m\lambda$, where $m$ is an integer and $\lambda$ the wavelength, reflected wavelengths interfere with each other and each wavelength component is diffracted at different angles according to

$$d(\sin \alpha + \sin \beta) = m\lambda,$$

where $\alpha$ is the angle of incidence and $\beta$ the angle of diffraction. Then, the angular dispersion is

$$\frac{d\beta}{d\lambda} = \frac{m}{d \cos \beta}.$$

Diffraction gratings operate as follows. When a polychromatic light beam impinges on a diffraction grating, all the wavelength components are diffracted at different angles. Knowing the wavelengths, the angle of incidence, and the specifications of the grating, one can calculate the angle of diffraction for each wavelength, and receiving fibers are placed at the focal points, a fiber for each wavelength (Figure 4.18).
Focusing the diffracted wavelengths may be achieved with a lens system or with a diffraction grating in a concave form.

Typically, diffraction gratings are produced by etching single silicon crystals.

**EXERCISES**

1. A Fabry–Perot interferometer has a refractive index $n = 1$. For $m = 1$, what should be the spacing to resonate at 1400 nm?

2. Calculate the finesse of a Fabry–Perot interferometer for two reflectivities:
   (a) $R = 0.9$.
   (b) $R = 0.3$.

3. The Bragg spacing is $5 \times 750$ nm. What is the order of the Bragg grating for a wavelength 1300 nm?

4. A Bragg grating has a grating constant $d = 700$ nm. For what wavelength value is the first-order reflection maximal?