II Civil Engineering
NUMERICAL ANALYSIS
Academic year 2013-2014

Exam Questions – February 2014

Item No.	*Chapter [– Sub-Chapter]:* Topic†
1	*Object of Numerical Analysis:* Object. Problem conditioning, condition number. Example (Hilbert matrix). Algorithm stability. Conclusions on problem conditioning and algorithm stability. Three-term recurrence formula.
2	*Computer representation of numbers – Floating-point representation:* Representation in the Model; Representation in a Binary Format. Normal & subnormal numbers; Overflow & underflow.
3	*Reals, floating-point representation:* IEEE Formats (Standard 754-2008 & 754-1985): Format structure; Format parameters.
4	*Computer representation of numbers:* Format encoding: Floating-Point data; Exponent encoding; maximum and minimum exponent. Significand encoding.
5	*Computer representation of numbers – IEEE Formats:* Special values: Format encoding; description.
6	*Computer representation of numbers – IEEE Formats:* Representation range (reals). Fortran intrinsic functions returning representation parameters.
7	*Computer representation of numbers – Rounding error measure:* ULP; Machine-ε; Unit rounding error.
8	*Errors, sources and propagation:* Error; Relative error. Significant digits; Correct representation with *m* significant digits; Relation with the relative error.
9	*Errors, sources and propagation:* Error sources. Rounding error; Truncation case. Wilkinson form of the error. Examples for base $\beta = 2$.
10	*Errors, sources and propagation – Error propagation:* Propagated error. Multiplication; Division; Function evaluation.
11	*Errors, sources and propagation – Error propagation:* Loss of signification error. Addition and subtraction. Propagation of errors in a sum.
12	*Errors, sources and propagation – Error propagation:* Summation (in Scientific computation); Examples. Conclusions regarding the numbers used in computation: the format needed for their representation.
13	*Nonlinear equations:* Method and method analysis. Order of convergence. Relation between absolute errors $\boldsymbol{e}_n$ and $\boldsymbol{e}_0$: case $p > 1$.
14	*Nonlinear equations – Order of convergence:* Relation between absolute errors $\boldsymbol{e}_n$ and $\boldsymbol{e}_0$: case $p = 1$. Linear convergence. Variant to convergence order; Asymptotic error constant.

Item No.	*Chapter [– Sub-Chapter]:* Topic†
15	*Roots of an equation f(x) = 0:* General considerations. Bisection method. Secant method; Remarks on secant method.
16	*Roots of an equation f(x) = 0 – Newton method:* Method; Convergence. Error estimation. Numerical evaluation of the derivative. Comparison with Secant method.
17	*Fixed-Point method:* Method. Convergence: contracting mapping; Theorems 1 & 2; Case g = differentiable; case $\lvert g'(x)\rvert > 1$.
18	*Fixed-Point method - Error propagation:* Isaacson & Keller Theorem (without proof); Evaluation of error $\delta/(1-\lambda)$; Conclusions; rational number of iterations.
19	*Fixed-Point method:* Higher order Fixed-point methods. Example: Newton method.
20	*Fixed-Point method – Implementation:* Error evaluation: Relation between $\lvert\boldsymbol{\alpha}-\boldsymbol{x_{n+1}}\rvert$ and $\lvert\boldsymbol{x_{n+1}}-\boldsymbol{x_n}\rvert$. *XTOL*-minimum evaluation. Algorithm: the divergence test.
21	*Fixed-Point method:* Geometrical interpretation. The stationary process (graphical interpretation).
22	*Fixed-Point method:* The stationary process of period 2; stationary process in general.
23	*Fixed-Point method:* Explicit Fixed-point procedures. Examples: $\Phi(x) = m$ = constant; Newton method.
24	*Multiple roots:* Definition; Computing problems; Newton method; Modified Newton method. Determination of the multiplicity order.
25	*Root of a polynomial:* Polynomial evaluation; Deflation; Newton method for polynomials.
26	*Root of a polynomial:* Strategies & Algorithms: Direct iteration (Pol_Direct-q & Pol_Direct-deriv). Deflation (Pol).
27	*Root of a polynomial:* Complex roots (elements); Laguerre Method; IMSL implementation. Stability of the roots.
28	*Systems of non-linear equations:* Definitions. Vector norm. Matrix norm; Matrix norm induced by vector norm; spectral radius.
29	*Systems of non-linear equations:* Fixed-point method. Convergence: main theorem (Theorem 2); jacobian. Second order convergence.
30	*Systems of non-linear equations:* Explicit fixed-point procedure; Iteration with constant matrix **A** (updated). Practical iteration scheme; iteration stopping tests.

Item No.	*Chapter [– Sub-Chapter]:* Topic[†]
31	*Systems of non-linear equations:* Newton Method; Convergence; Practical iteration scheme; iteration stopping tests. Newton-like methods.
32	*Linear systems of equations:* General considerations. Gauss elimination: method; pivoting (concept). Triangular factorization of system matrix; determinant computation.
33	*Linear systems of equations – Gauss elimination:* Number of operations; Comparison with other processes. Matrix inversion, number of operations.
34	*Linear systems of equations – Gauss elimination:* Pivoting in Gauss elimination: partial & complete pivoting. Solution steps by **LU** decomposition; Number of operations.
35	*Linear systems of equations:* Direct evaluation of **LU** factors; methods. Possibility of **LU** factorization; pivoting.
36	*Linear systems of equations – Cholesky method:* Symmetric and positive definite matrices: definition, properties. Cholesky method; solution steps; number of operations; Factorization without square roots computation.
37	*Error analysis & Solution stability:* Perturbation in the RHS **b**. Condition number; properties; matrix conditioning. Condition number $Cond(\mathbf{A})_{*}$; Computation formula. Example of ill-conditioned matrix.

[†] *Chapter/Sub-chapter* and Topic refer to the content taught in Course lectures & Lab classes.

January 17, 2014

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