Structura matricii de masă

1. Mase concentrate

Ecuaţia de mişcare este (1) – v. ”Determinarea pulsaţiilor proprii ...”:

$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{f}(\mathbf{U}) = \mathbf{P}(t)$,

Pentru analiza dinamică liniară, ecuaţia anterioară se aduce la forma (7)

$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{0}$.

Ecuaţia matriceală (1) provine din ecuaţiile de mişcare de nod, anume

$$\frac{\overline{\rho}^0}{6}\left[\sum_J \Delta s^0_{K-1,J}\ddot{\mathbf{U}}_{K-1,J} + \left(\sum_J 2(\Delta s^0_{K-1,J} + \Delta s^0_{K,J})\right)\ddot{\mathbf{U}}_K + \sum_J \Delta s^0_{K+1,J}\ddot{\mathbf{U}}_{K+1,J}\right] + \overline{m}_K\ddot{\mathbf{U}}_K$$
$$+ \sum_J (T_{K-1,J}\mathbf{V}_{K-1,J} - T_{K,J}\mathbf{V}_{K,J})$$
$$-\mathbf{P}_K = \mathbf{0}$$

Vezi “Deducerea ecuaţiilor de mişcare” § 5.2. $\overline{m}_K$ este masa concentrată în nodul K

Pentru modelul de mase concentrate, ecuaţia de mişcare de nod se obţine punând în ecuaţia anterioară $\overline{\rho}^0 = 0$. Rezultă

$\overline{m}_K\ddot{\mathbf{U}}_K + \mathbf{f}_K(\mathbf{U}) - \mathbf{P}_K = \mathbf{0}$,

în care

$\mathbf{f}_K(\mathbf{U}) = \sum_J (T_{K-1,J}\mathbf{V}_{K-1,J} - T_{K,J}\mathbf{V}_{K,J})$.

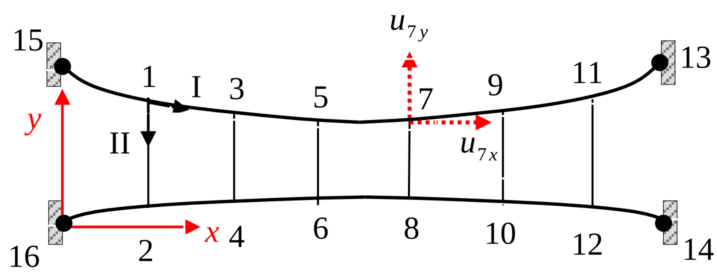
Sau, notând acum m_K în loc de $\overline{m}_K$, avem:

$m_K\ddot{\mathbf{U}}_K + \mathbf{f}_K(\mathbf{U}) - \mathbf{P}_K = \mathbf{0}$.

Considerăm o structură cu 2 grade de libertate de nod – de exemplu, o fermă-cablu. Notăm numărul nodurilor cu N: numărul total de grade de libertate este $2 \times N$.

Exemplu:

Pentru ferma-cablu din figura de mai jos: $N = 12$; numărul gradelor de libertate este $2N = 24$ ■



Vectorul deplasărilor de nod este

$$\mathbf{U} = \begin{bmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \\ \vdots \\ \mathbf{U}_N \end{bmatrix} = \begin{bmatrix} u_{1,x} \\ u_{1,y} \\ u_{2,x} \\ u_{2,y} \\ \vdots \\ u_{N,x} \\ u_{N,y} \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ \vdots \\ u_{2N-1} \\ u_{2N} \end{bmatrix}$$

Ecuaţia de mişcare a nodului *K*, devine:

$$m_K \begin{bmatrix} \ddot{u}_{K,x} \\ \ddot{u}_{K,y} \end{bmatrix} + \begin{bmatrix} f_{K,x}(\mathbf{U}) \\ f_{K,y}(\mathbf{U}) \end{bmatrix} - \begin{bmatrix} P_{K,x} \\ P_{K,y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Sau, cu numerotarea generală

$$m_K \begin{bmatrix} \ddot{u}_{2K-1} \\ \ddot{u}_{2K} \end{bmatrix} + \begin{bmatrix} f_{2K-1}(\mathbf{U}) \\ f_{2K}(\mathbf{U}) \end{bmatrix} - \begin{bmatrix} P_{2K-1} \\ P_{2K} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Combinând ecuaţiile anterioare, pentru $K = 1,2,\ldots,N$, rezultă:

$$\begin{array}{l} 1,x: \\ 1,y: \\ 2,x: \\ 2,y: \\ \vdots\,\vdots \\ N,x: \\ N,y: \end{array} \begin{bmatrix} m_1 &&&&&& \\ & m_1 &&&&& \\ && m_2 &&&& \\ &&& m_2 &&& \\ &&&& \ddots && \\ &&&&& m_N & \\ &&&&&& m_N \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ \vdots \\ u_{2N-1} \\ u_{2N} \end{bmatrix} + \cdots = \mathbf{0}$$

Astfel, structura matricii de masă este

$$\mathbf{M}=\begin{bmatrix} m_1 & & & & & & \\ & m_1 & & & & & \\ & & m_2 & & & & \\ & & & m_2 & & & \\ & & & & \ddots & & \\ & & & & & m_N & \\ & & & & & & m_N \end{bmatrix}$$

Pentru o structură cu 3 grade de libertate/nod, matricea de masă va avea structura

$$\mathbf{M}=\begin{bmatrix} m_1 & & & & & & & & & \\ & m_1 & & & & & & & & \\ & & m_1 & & & & & & & \\ & & & m_2 & & & & & & \\ & & & & m_2 & & & & & \\ & & & & & m_2 & & & & \\ & & & & & & \ddots & & & \\ & & & & & & & m_N & & \\ & & & & & & & & m_N & \\ & & & & & & & & & m_N \end{bmatrix}$$

2. Mase distribuite

Matricea de masă va avea o structură de matrice bandă, similară cu cea a matricii de rigiditate.

Ea va fi furnizată de programul DINSAS.

Luăm ca exemplu ferma-cablu din Exemplul 2.1 – ”Determinarea pulsaţiilor proprii ...”.

Ferma are 18 noduri şi 36 grade de libertate

Considerând masele distribuite pe cabluri, cu densitatea de masă (volumică) $\rho^0 = 0.102\text{E-}05$ kgf/cm/sec2/(cm*cm2), această matrice este:

Mass Matrix (Upper Triangle) (kgf*s^2/cm); Half-Bandwidth: 6

2.8323930E-05 0.000000 5.6447475E-06 0.000000 4.4293820E-06 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000 0.000000 0.000000

2.8323930E-05 0.000000 5.6447475E-06 0.000000 4.4293820E-06 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000 0.000000

2.8644912E-05 0.000000 0.000000 0.000000 4.3126424E-06 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000

2.8644912E-05 0.000000 0.000000 0.000000 4.3126424E-06 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000

2.5480278E-05 0.000000 3.9950623E-06 0.000000 4.3156947E-06 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000

2.5480278E-05 0.000000 3.9950623E-06 0.000000 4.3156947E-06 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000

2.5289957E-05 0.000000 0.000000 0.000000 4.3372747E-06 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000 0.000000 0.000000

2.5289957E-05 0.000000 0.000000 0.000000 4.3372747E-06 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000 0.000000

2.3497878E-05 0.000000 3.0600970E-06 0.000000 4.3731475E-06 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000

2.3497878E-05 0.000000 3.0600970E-06 0.000000 4.3731475E-06 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
0.000000

2.3505079E-05 0.000000 0.000000 0.000000 4.3551677E-06 0.000000

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 0.000000        0.000000        0.000000        0.000000        0.000000        0.000000
 0.000000        0.000000        0.000000        0.000000        0.000000        0.000000

2.3505079E-05    0.000000        0.000000        0.000000     4.3551677E-06    0.000000
 0.000000        0.000000        0.000000        0.000000        0.000000        0.000000
 0.000000        0.000000        0.000000        0.000000        0.000000

2.2697672E-05    0.000000     2.7031192E-06    0.000000     4.2725687E-06    0.000000
 0.000000        0.000000        0.000000        0.000000        0.000000        0.000000
 0.000000        0.000000        0.000000        0.000000

2.2697672E-05    0.000000     2.7031192E-06    0.000000     4.2725687E-06    0.000000
 0.000000        0.000000        0.000000        0.000000        0.000000        0.000000
 0.000000        0.000000        0.000000

2.2623402E-05    0.000000        0.000000        0.000000     4.2534143E-06    0.000000
 0.000000        0.000000        0.000000        0.000000        0.000000        0.000000
 0.000000        0.000000

2.2623402E-05    0.000000        0.000000        0.000000     4.2534143E-06    0.000000
 0.000000        0.000000        0.000000        0.000000        0.000000        0.000000
 0.000000

2.3423911E-05    0.000000     3.0938536E-06    0.000000     4.3455334E-06    0.000000
 0.000000        0.000000        0.000000        0.000000        0.000000        0.000000

2.3423911E-05    0.000000     3.0938536E-06    0.000000     4.3455334E-06    0.000000
 0.000000        0.000000        0.000000        0.000000        0.000000

2.3403070E-05    0.000000        0.000000        0.000000     4.3542673E-06    0.000000
 0.000000        0.000000        0.000000        0.000000

2.3403070E-05    0.000000        0.000000        0.000000     4.3542673E-06    0.000000
 0.000000        0.000000        0.000000

2.5281317E-05    0.000000     3.9949505E-06    0.000000     4.3001755E-06    0.000000
 0.000000        0.000000

2.5281317E-05    0.000000     3.9949505E-06    0.000000     4.3001755E-06    0.000000
 0.000000

2.5767076E-05    0.000000        0.000000        0.000000     4.5343209E-06    0.000000

2.5767076E-05    0.000000        0.000000        0.000000     4.5343209E-06

2.8504488E-05    0.000000     5.5137557E-06    0.000000

2.8504488E-05    0.000000     5.5137557E-06

2.8256833E-05    0.000000

2.8256833E-05
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Notă: Semi-laţimea de bandă raportată (*LIM*), este cea a matricii de rigiditate. ■