

PROBLEM No. 1 / II

Given a real number $x > 0$, let $y(x)$ be the smallest (positive) number, that added to x gives a result which does not round to x (is greater than x).

Write a program which determine $y(x)$ for $x = 2, 3, \dots, 18$ (step 1), and verify that $x + y(x) > x$.

PROBLEM No. 2 / II

Calculate the values of the following functions, at values $x = 10^i$, $i = 1, 2, \dots, 7$.

$$f(x) = x(\sqrt{x^2 + 4} - \sqrt{x^2 - 3}) \quad - \text{ In single precision;}$$

$$f_2(x) = x(\sqrt{x^2 + 4} - \sqrt{x^2 - 3}) \quad - \text{ In double precision;}$$

$$g(x) = \frac{7x}{\sqrt{x^2 + 1} + \sqrt{x^2 - 3}} \quad - \text{ In single precision.}$$

- Tabulate computed values ($f_2(x)$ with 15 significant digits), and explain the results.
- Find out the number of correct significant digits of $f(x)$ value for $x = 10^3$.

PROBLEM No. 3 / II

Equation $f(x) = e^x - x^4 = 0$ has two roots.

- Find intervals of length 1, containing the roots.
- Find the roots by BISECTION method, with tolerance $EPS = 10^{-6}$.
- Find again the roots, by SECANT method, with the same tolerance, and compare the number of iterations.

PROBLEM No. 4 / II

Given the equation $f(x) = 0$, where:

$$f(x) = 0.9 \cos(x) - x + 1.6$$

- 1) Solve the equation by NEWTON method, taking $x_0 = 1.5$ and tolerance $EPS = 10^{-6}$.
- 2) Solve the equation by SECANT method, with $x_0 = 1.5$, $x_1 = 1.6$, and $EPS = 10^{-6}$.
- 3) Compare the number of iterations in the two methods.

PROBLEM No. 5 / II

Given the equation:

$$tg(x) = \frac{1.82 - x}{x - 0.2}$$

- 1) Solve the equation by NEWTON method, choosing $x_0 = 0.8$ and tolerance

$$EPS = 10^{-6}.$$

- 2) Solve the equation by SECANT method, with $x_0 = 0.7$, $x_1 = 0.9$, and

$$EPS = 10^{-6}.$$

- 3) Compare the number of iterations in the two methods.

PROBLEM No. 6 / II

Consider the function

$$f(x) = -x + e^{-px^3} \cos(x),$$

where p is a parameter.

The equation $f(x) = 0$ has a unique root in the interval $(0, 1)$.

- 1) Find the roots, with a tolerance $EPS = 10^{-6}$, for the following p values: $p = 1$;

5; and 25.

- 2) Solve by Newton method the case $p = 25$, with $EPS = 10^{-6}$ and $x_0 = 0$.

- 3) Comment the result.

PROBLEM No. 7 / II

Equation $e^x - 2x^2 = 0$ has 3 roots.

For the positive roots, consider the equation put in the form $x = g(x)$, where:

1) $g(x) = \sqrt{e^x / 2}$.

Iterate (in the fixed-point method), with $x_0 = 0.5$ for the first root, and with

$x_0 = 2.5$ for the second root. Use the tolerance $XTOL = 10^{-6}$.

2) $g(x) = x - m(e^x - 4x^2)$

For the root near $x_0 = 2.5$:

- Set up m so that the process converge, and compute the root.
- Determine the value of m for which the iteration converges the most rapidly.

PROBLEM No. 8 / II

Consider the equation $x = g(x)$, where

$$g(x) = 0.61 + 1.07 \cos(x)$$

1) Iterate in single precision, with $x_0 = 1.2$, tolerance $EPS = 10^{-6}$, and limited number of iterations $NLIM \geq 300$. Comment the result.

2) Repeat the iteration in double precision and find the root with tolerance $EPS = 10^{-9}$.

PROBLEM No. 9 / II

Given the annuity equation

$$P_1[(1+r)^{N_1} - 1] = P_2[1 - (1+r)^{-N_2}],$$

where: r = yearly nominal interest rate; P_1 = amount of deposit at the beginning of years 1, 2, ..., N_1 ; P_2 = amount of the payment at the beginning of years

$N_1 + 1, N_2 + 1, \dots, N_1 + N_2$. After the last payment, the account balance is zero.

Find r for values: $N_1 = 35$, $N_2 = 25$, $P_1 = 5000$, and $P_2 = 10000$, by:

- a) Newton method;
- b) Secant method.

PROBLEM No. 10 / II

The Redlich-Kwong state equation (state equation with two parameters, of a real gas) is:

$$P = \frac{RT}{v-b} - \frac{a}{T^{1/2}v(v+b)}$$

For values: $P = 100.3 \times 10^5$, $T = 673.15$, $R = 461.49$; and, $a = 43890.21$, $b =$

1.17043×10^{-3} , determinate the value v from the equation, by a numerical method.

Hint: The magnitude order of v is 10^{-2} .

[P = pressure (Pa); T = temperature (K); R = ideal gas constant (J/(kg³K)); v = specific volume (m³/kg); a (m⁵K^{0.5}/(kg³s²)) and b (m³/kg) are empirical constants.

Numerical values refer to steam.]

PROBLEM No. 11 / II

In the problem of missile interception, the following system is obtained:

$$\begin{cases} t \cos(\alpha) + t - 1 = 0 \\ t \sin(\alpha) - 0.1t^2 + e^{-t} - 1 = 0 \end{cases}$$

(t is the time, and α is the firing angle of the interceptor.)

Solve the system, with tolerance $EPS = 10^{-6}$ and initial approximation

$$w_0 = (0.5, 1), \text{ by:}$$

- 1) Iteration with constant matrix A (with updating after 3 steps)
- 2) NEWTON method.

PROBLEM No. 12 / II

Consider the system:

$$\begin{cases} x^2 + 2 \sin(y) + z = -0.1 \\ \cos(y) - z = 2.1 \\ x^2 + y^2 + z^2 = 2 \end{cases}$$

Find the solution near $w_0 = (1, 0, -1)$, with tolerance $EPS = 10^{-6}$, either by

Newton method, or by iteration with constant matrix A (with updating after 3 steps).

PROBLEM No. 13 / II

Given the CHEBISHEV polynomial of 2nd kind, of order 6:

$$T_6(x) = 64x^6 - 80x^4 + 24x^2 - 1.$$

- 1) Find initial approximations of the roots (by algebraic methods, function graph, etc.)
- 2) Calculate the roots, with tolerance 10^{-6} .
- 3) *Note:* All roots are of modulus < 1 .

PROBLEM No. 14 / II

The polynomial

$$p(x) = x^5 - 18x^4 + 118x^3 - 348x^2 + 457x - 210$$

Has the roots: $x_1 = 1$; $x_2 = 2$; $x_3 = 3$; $x_4 = 5$; $x_5 = 7$.

Let $\tilde{p}(x)$ be the polynomial obtained from $p(x)$ by replacing the coefficient

$a_3 = 118$ of x^3 , with $\tilde{a}_3 = 118.02$.

- Calculate the roots of $\tilde{p}(x)$.
- Calculate the ratio: maximum relative perturbation in the roots (in modulus) / the relative perturbation in the coefficient a_3 . Comment the result.
- (If a perturbed becomes \tilde{a} , the relative perturbation is: $(\tilde{a} - a) / a$.)

PROBLEM No. 15 / II

Consider the system:

$$\begin{cases} (x - y)(x + y)^{1/2} = 3 \\ x - \log(x - y) = 1 \end{cases}$$

Solve the system, by iteration with constant matrix A (updated after 3 steps), with:

- Tolerance $EPS = 10^{-6}$.
- Initial approximation $w_0 = (2, -0.5)$.

PROBLEM No. 16 / II

Consider the system:

$$\begin{cases} \sin(x + y) = x + 0.1 \\ \cos(x - y) = y + 0.5 \end{cases}$$

- Solve the system by NEWTON method, with tolerance $EPS = 10^{-6}$: Find the root near $w_0 = (1, 0)$
- Solve again, by iteration with constant matrix A (with updating after 3 steps).

PROBLEM No. 17 / II

Consider the system:

$$\begin{cases} \sin(xy) = 0.5 \\ \cos(x) = e^y \end{cases}$$

- Solve by NEWTON method, with tolerance $EPS = 10^{-6}$: Find the roots near $w_0 = (-1, -0.5)$, and $w_0 = (5, -1)$, respectively
- Solve again, by iteration with constant matrix A (with updating after 3 steps).

PROBLEM No. 18 / II

Consider the system:

$$\begin{cases} f_1(x, y) = x^2 + 4y^2 - 16 = 0 \\ f_2(x, y) = -x^2 + y + 3 = 0 \end{cases}$$

Solve by NEWTON method, with tolerance $EPS = 10^{-6}$. Find the initial approximations x_0, y_0 , from the intersection of the curves $f_1(x, y) = 0$ and $f_2(x, y) = 0$.

PROBLEM No. 19 / II

Consider the system:

$$\begin{cases} x + y + z^2 = 3.8 \\ xyz = -1.9 \\ \sqrt{x} + \sqrt{y} - z^2 = 1.3 \end{cases}$$

By iteration with constant matrix A (updated after 3 steps), find the root near

$$w_0 = (1.5, 1, -1), \text{ with a tolerance } EPS = 10^{-6}.$$

PROBLEM No. 20 / II

Consider the linear system $Ax = b$, where:

$$A = \begin{bmatrix} 2 & 3 & 4 & 5.01 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 5.01 & 6 & 7 & 8 \end{bmatrix}; \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

1) Calculate the solution for the right-hand sides $b = [1 \ 2 \ 3 \ 4]^T$, and

$$\tilde{b} = [1 \ 2 \ 3 \ 4.01]^T$$

2) Calculate the ratio: maximum relative perturbation (in modulus) in solution / relative perturbation in the right-hand side b_4 . Comment the result.

PROBLEM No. 21 / II

Given the matrix:

$$A = \begin{bmatrix} 2 & 3 & 4 & 5.01 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 5.01 & 6 & 7 & 8 \end{bmatrix}$$

- 1) Calculate the inverse A^{-1} of A
- 2) Calculate the numbers of condition $cond(A)_1$ and $cond(A)_\infty$.

PROBLEM No. 22 / II

Establish if the matrix

$$A = \begin{bmatrix} 5 & 6 & 7 & 8.01 \\ 6 & 7 & 8 & 9 \\ 7 & 8 & 9 & 10 \\ 8.01 & 9 & 10 & 11 \end{bmatrix}$$

is well- or ill-conditioned.

PROBLEM No. 23 / II

Consider the HILBERT matrix of order 4:

$$H_4 = \begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 \\ 1/2 & 1/3 & 1/4 & 1/5 \\ 1/3 & 1/4 & 1/5 & 1/6 \\ 1/4 & 1/5 & 1/6 & 1/7 \end{bmatrix}$$

(Matrix elements are: $h_{ij} = 1/(i + j - 1)$)

- 1) Calculate H_4^{-1} , in single precision.
- 2) Calculate the number of condition $cond(H_4)_1$.

PROBLEM No. 24 / II

Given the HILBERT matrix of order 4:

$$H_4 = \begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 \\ 1/2 & 1/3 & 1/4 & 1/5 \\ 1/3 & 1/4 & 1/5 & 1/6 \\ 1/4 & 1/5 & 1/6 & 1/7 \end{bmatrix}$$

- 1) Solve the linear system $H_4 x = b$, for:

$$b = [1 \quad 0.2 \quad 0.3 \quad 0.4]^T, \text{ and } b = [1.02 \quad 0.2 \quad 0.3 \quad 0.4]^T.$$

- 2) Calculate the ratio: the maximum relative perturbation (in modulus) in solution / the relative perturbation in the right-hand side b_1 (in modulus).

Comment the result.

PROBLEM No. 25 / II

Consider the linear system $Ax = b$, where:

$$A = \begin{bmatrix} 5 & -1 & -1 & 0 & 0 \\ -1 & 5 & -1 & -1 & 0 \\ -1 & -1 & 5 & -1 & -1 \\ 0 & -1 & -1 & 5 & -1 \\ 0 & 0 & -1 & -1 & 5 \end{bmatrix}, \quad b = \begin{bmatrix} -1 & | & 3 \\ 1 & | & 2 \\ -1 & | & 1 \\ 1 & | & 2 \\ -1 & | & 3 \end{bmatrix}$$

Matrix A is positive definite.

- 1) Solve the system by CHOLESKY method.
- 2) Display matrix L , and calculate the determinant of matrix A .

PROBLEM No. 26 / II

Consider the linear system $Ax = b$, where:

$$A = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 4 & 3 \\ 2 & 3 & 4 \end{bmatrix}, \quad b = \begin{bmatrix} 1 & | & 7 \\ 1 & | & 8 \\ 1 & | & 9 \end{bmatrix},$$

and matrix A is positive definite.

- 1) Solve the system by CHOLESKY method.
- 2) Display matrix L , and calculate the determinant of A .

PROBLEM No. 27 / II

Given the matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$$

- 1) Calculate the numbers of condition $cond(A)_1$ and $cond(A)_\infty$.
- 2) Calculate the number of condition $cond(A)_*$.

PROBLEM No. 28 / II

Given the matrix:

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 2 & 0.5 \\ 1 & 0.5 & 4 \end{bmatrix}$$

- Calculate the number of condition $cond(A)_*$.
- Is the matrix well- or ill-conditioned?