PROBLEM No. 1 / II

Given a real number $x > 0$, let $y(x)$ be the smallest (positive) number, that added to x gives a result which does not round to x (is greater than x).

Write a program which determine $y(x)$ for $x = 2, 3, \ldots, 18$ (step 1), and verify that $x + y(x) > x$.

PROBLEM No. 2 / II

Calculate the values of the following functions, at values $x = 10^i$, $i = 1, 2, \ldots, 7$.

$f(x) = x(\sqrt{x^2+4} - \sqrt{x^2-3})$ - In single precision;

$f2(x) = x(\sqrt{x^2+4} - \sqrt{x^2-3})$ - In double precision;

$g(x) = \dfrac{7x}{\sqrt{x^2+1} + \sqrt{x^2-3}}$ - In single precision.

- Tabulate computed values ($f_2(x)$ with 15 significant digits), and explain the results.
- Find out the number of correct significant digits of $f(x)$ value for $x = 10^3$.

PROBLEM No. 3 / II

Equation $f(x) = e^x - x^4 = 0$ has two roots.

- Find intervals of length 1, containing the roots.
- Find the roots by BISECTION method, with tolerance $EPS = 10^{-6}$.
- Find again the roots, by SECANT method, with the same tolerance, and compare the number of iterations.

PROBLEM No. 4 / II

Given the equation $f(x) = 0$, where:

$f(x) = 0.9\cos(x) - x + 1.6$

1) Solve the equation by NEWTON method, taking $x_0 = 1.5$ and tolerance $EPS = 10^{-6}$.
2) Solve the equation by SECANT method, with $x_0 = 1.5$, $x_1 = 1.6$, and $EPS = 10^{-6}$.
3) Compare the number of iterations in the two methods.

PROBLEM No. 5 / II

Given the equation:

$$tg(x) = \frac{1.82 - x}{x - 0.2}$$

1) Solve the equation by NEWTON method, choosing $x_0 = 0.8$ and tolerance $EPS = 10^{-6}$.
2) Solve the equation by SECANT method, with $x_0 = 0.7$, $x_1 = 0.9$, and $EPS = 10^{-6}$.
3) Compare the number of iterations in the two methods.

PROBLEM No. 6 / II

Consider the function

$$f(x) = -x + e^{-px^3}\cos(x),$$

where p is a parameter.

The equation $f(x) = 0$ has a unique root in the interval $(0, 1)$.

1) Find the roots, with a tolerance $EPS = 10^{-6}$, for the following p values: p = 1; 5; and 25.
2) Solve by Newton method the case $p = 25$, with $EPS = 10^{-6}$ and $x_0 = 0$.
3) Comment the result.

PROBLEM No. 7 / II

Equation $e^x - 2x^2 = 0$ has 3 roots.

For the positive roots, consider the equation put in the form $x = g(x)$, where:

1) $g(x) = \sqrt{e^x / 2}$.

Iterate (in the fixed-point method), with $x_0 = 0.5$ for the first root, and with $x_0 = 2.5$ for the second root. Use the tolerance $XTOL = 10^{-6}$.

2) $g(x) = x - m(e^x - 4x^2)$

For the root near $x_0 = 2.5$:

- Set up *m* so that the process converge, and compute the root.
- Determine the value of *m* for which the iteration converges the most rapidly.

PROBLEM No. 8 / II

Consider the equation $x = g(x)$, where

$g(x) = 0.61 + 1.07\cos(x)$

1) Iterate in single precision, with $x_0 = 1.2$, tolerance $EPS = 10^{-6}$, and limited number of iterations $NLIM \geq 300$. Comment the result.
2) Repeat the iteration in double precision and find the root with tolerance $EPS = 10^{-9}$.

PROBLEM No. 9 / II

Given the annuity equation

$$P_1[(1+r)^{N_1} - 1] = P_2[1-(1+r)^{-N_2}],$$

where: r = yearly nominal interest rate; P_1 = amount of deposit at the beginning of years $1, 2, \ldots, N_1$; P_2 = amount of the payment at the beginning of years $N_1+1, N_2+1, \ldots, N_1+N_2$. After the last payment, the account balance is zero.

Find r for values: $N_1 = 35$, $N_2 = 25$, $P_1 = 5000$, and $P_2 = 10000$, by:

a) Newton method;

b) Secant method.

PROBLEM No. 10 / II

The Redlich-Kwong state equation (state equation with two parameters, of a real gas) is:

$$P = \frac{RT}{v-b} - \frac{a}{T^{1/2}v(v+b)}$$

For values: P = 100.3×10^5, T = 673.15, R = 461.49; and, a = 43890.21, b = 1.17043×10^{-3}, determinate the value v from the equation, by a numerical method.

Hint: The magnitude order of v is 10^{-2}.

[P = pressure (Pa); T = temperature (K); R = ideal gas constant (J/(kg^{3}K)); v = specific volume (m^3/kg); a (m^{5}K$^{0.5}$/(kg^{3}s^2)) and b (m^3/kg) are empirical constants. Numerical values refer to steam.]

PROBLEM No. 11 / II

In the problem of missile interception, the following system is obtained:

$$\begin{cases} t\cos(\alpha) + t - 1 = 0 \\ t\sin(\alpha) - 0.1t^2 + e^{-t} - 1 = 0 \end{cases}$$

(t is the time, and α is the firing angle of the interceptor.)

Solve the system, with tolerance $EPS = 10^{-6}$ and initial approximation

$w_0 = (0.5, 1)$, by:

1) Iteration with constant matrix A (with updating after 3 steps)

2) NEWTON method.

PROBLEM No. 12 / II

Consider the system:

$$\begin{cases} x^2 + 2\sin(y) + z = -0.1 \\ \cos(y) - z = 2.1 \\ x^2 + y^2 + z^2 = 2 \end{cases}$$

Find the solution near $w_0 = (1, 0, -1)$, with tolerance $EPS = 10^{-6}$, either by Newton method, or by iteration with constant matrix A (with updating after 3 steps).

PROBLEM No. 13 / II

Given the CHEBISHEV polynomial of 2nd kind, of order 6:

$T_6(x) = 64x^6 - 80x^4 + 24x^2 - 1.$

1) Find initial approximations of the roots (by algebraic methods, function graph, etc.)
2) Calculate the roots, with tolerance 10^{-6}.
3) *Note*: All roots are of modulus < 1.

PROBLEM No. 14 / II

The polynomial

$p(x) = x^5 - 18x^4 + 118x^3 - 348x^2 + 457x - 210$

Has the roots: $x_1 = 1;\ x_2 = 2;\ x_3 = 3;\ x_4 = 5;\ x_5 = 7$.

Let $\tilde{p}(x)$ be the polynomial obtained form $p(x)$ by replacing the coefficient

$a_3 = 118$ of x^3, with $\tilde{a}_3 = 118.02$.

- Calculate the roots of $\tilde{p}(x)$.
- Calculate the ratio: maximum relative perturbation in the roots (in modulus) / the relative perturbation in the coefficient a_3. Comment the result.
- (If *a* perturbed becomes $\tilde{a}$, the relative perturbation is: $(\tilde{a} - a)/a$.)

PROBLEM No. 15 / II

Consider the system:

$$\begin{cases} (x-y)(x+y)^{1/2} = 3 \\ x - \log(x-y) = 1 \end{cases}$$

Solve the system, by iteration with constant matrix A (updated after 3 steps), with:

- Tolerance $EPS = 10^{-6}$.
- Initial approximation $w_0 = (2, -0.5)$.

PROBLEM No. 16 / II

Consider the system:

$$\begin{cases} \sin(x+y) = x + 0.1 \\ \cos(x-y) = y + 0.5 \end{cases}$$

- Solve the system by NEWTON method, with tolerance $EPS = 10^{-6}$: Find the root near $w_0 = (1, 0)$
- Solve again, by iteration with constant matrix A (with updating after 3 steps).

PROBLEM No. 17 / II

Consider the system:

$$\begin{cases} \sin(xy) = 0.5 \\ \cos(x) = e^{y} \end{cases}$$

- Solve by NEWTON method, with tolerance $EPS = 10^{-6}$: Find the roots near $w_0 = (-1, -0.5)$, and $w_0 = (5, -1)$, respectively
- Solve again, by iteration with constant matrix A (with updating after 3 steps).

PROBLEM No. 18 / II

Consider the system:

$$\begin{cases} f_1(x, y) = x^2 + 4y^2 - 16 = 0 \\ f_2(x, y) = -x^2 + y + 3 = 0 \end{cases}$$

Solve by NEWTON method, with tolerance $EPS = 10^{-6}$. Find the initial approximations x_0, y_0, from the intersection of the curves $f_1(x, y) = 0$ and $f_2(x, y) = 0$.

PROBLEM No. 19 / II

Consider the system:

$$\begin{cases} x + y + z^2 = 3.8 \\ xyz = -1.9 \\ \sqrt{x} + \sqrt{y} - z^2 = 1.3 \end{cases}$$

By iteration with constant matrix A (updated after 3 steps), find the root near

$w_0 = (1.5, 1, -1)$, with a tolerance $EPS = 10^{-6}$.

PROBLEM No. 20 / II

Consider the linear system $Ax = b$, where:

$$A = \begin{bmatrix} 2 & 3 & 4 & 5.01 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 5.01 & 6 & 7 & 8 \end{bmatrix}; \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

1) Calculate the solution for the right-hand sides $b = [1 \quad 2 \quad 3 \quad 4]^T$, and

   $\tilde{b} = [1 \quad 2 \quad 3 \quad 4.01]^T$

2) Calculate the ratio: maximum relative perturbation (in modulus) in solution / relative perturbation in the right-hand side b_4. Comment the result.

PROBLEM No. 21 / II

Given the matrix:

$$A = \begin{bmatrix} 2 & 3 & 4 & 5.01 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 5.01 & 6 & 7 & 8 \end{bmatrix}$$

1) Calculate the inverse A^{-1} of A

2) Calculate the number of condition $cond(A)_\infty = \| \mathbf{A} \|_\infty \cdot \| \mathbf{A}^{-1} \|_\infty$.

PROBLEM No. 22 / II

Given the matrix

$$A = \begin{bmatrix} 5 & 6 & 7 & 8.01 \\ 6 & 7 & 8 & 9 \\ 7 & 8 & 9 & 10 \\ 8.01 & 9 & 10 & 11 \end{bmatrix}$$

1) Calculate the inverse A^{-1} of A.

2) Calculate the number of condition $cond(A)_1 = \| \mathbf{A} \|_1 \cdot \| \mathbf{A}^{-1} \|_1$.

PROBLEM No. 23 / II

Consider the HILBERT matrix of order 4:

$$H_4 = \begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 \\ 1/2 & 1/3 & 1/4 & 1/5 \\ 1/3 & 1/4 & 1/5 & 1/6 \\ 1/4 & 1/5 & 1/6 & 1/7 \end{bmatrix}$$

(Matrix elements are: $h_{ij} = 1./(i + j - 1)$)

1) Calculate H_4^{-1}, in single precision.

2) Calculate the number of condition $cond(A)_1 = \| \mathbf{A} \|_1 \cdot \| \mathbf{A}^{-1} \|_1$.

PROBLEM No. 24 / II

Given the HILBERT matrix of order 4:

$$H_4 = \begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 \\ 1/2 & 1/3 & 1/4 & 1/5 \\ 1/3 & 1/4 & 1/5 & 1/6 \\ 1/4 & 1/5 & 1/6 & 1/7 \end{bmatrix}$$

1) Solve the linear system $H_4 x = b$, for:

   $b = [1 \quad 0.2 \quad 0.3 \quad 0.4]^T$, and $b = [1.02 \quad 0.2 \quad 0.3 \quad 0.4]^T$.

2) Calculate the ratio: the maximum relative perturbation (in modulus) in solution / the relative perturbation in the right-hand side b_1 (in modulus). Comment the result.

PROBLEM No. 25 / II

Consider the linear system $Ax = b$, where:

$$A = \begin{bmatrix} 5 & -1 & -1 & 0 & 0 \\ -1 & 5 & -1 & -1 & 0 \\ -1 & -1 & 5 & -1 & -1 \\ 0 & -1 & -1 & 5 & -1 \\ 0 & 0 & -1 & -1 & 5 \end{bmatrix}, \qquad b = \left[\begin{array}{c:c} -1 & 3 \\ 1 & 2 \\ -1 & 1 \\ 1 & 2 \\ -1 & 3 \end{array}\right]$$

Matrix *A* is positive definite.

1) Solve the system by CHOLESKY method.

2) Display matrix *L*, and calculate the determinant of matrix *A*.

PROBLEM No. 26 / II

Consider the linear system $Ax = b$, where:

$$A = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 4 & 3 \\ 2 & 3 & 4 \end{bmatrix}, \qquad b = \left[\begin{array}{c:c} 1 & 7 \\ 1 & 8 \\ 1 & 9 \end{array}\right],$$

and matrix *A* is positive definite.

1) Solve the system by CHOLESKY method.

2) Display matrix *L*, and calculate the determinant of *A*.